A Refinement of the Classical Order Point Model

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ABSTRACT

Factors such as demand volume and replenishment lead time that influence production and inventory control systems are random variables. Existing inventory models incorporate the parameters (e.g., mean and standard deviation) of these statistical quantities to formulate inventory policies. In practice, only sample estimates of these parameters are available. The estimates are subject to sampling variation and hence are random variables. Whereas the effect of sampling variability on estimates of parameters are in general well known in statistics literature, literature on inventory control policies has largely ignored the potential effect of sampling variation on the validity of the inventory models. This paper investigates the theoretical effect of sampling variability and develops theoretically sound inventory models that can be effectively used in different inventory policies.

Keywords: *Inventory Control, Inventory Management, Inventory Models, Order Point, Safety Stock*

INTRODUCTION

The main objectives of this paper are to: (1) critically review the basic traditional inventory model (S, Q), and (2) propose improved and relatively simple formulations for the (s, Q) model that can be used by practitioners. The improved formulation produces service levels that are on average equal to the desired or intended service levels. In addition, the methodology may be extended to other common inventory models (e.g., R, S model) with relative ease.

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Graves (1988) and Silver, Pyke, and Peterson (1998) recognized that stock are generated because of different circumstances or are kept for a variety of purposes without ear-marking them. A component of inventory, *cycle stock*, is generated to achieve economies of scale through batch production, batch material handling or transportation (Tsou, 2009). Technical issues are another reason for batch production, especially in process industries. *Pipeline stock* or *work-in-process* inventory, is generated because of processing or transfer lead time (Pettersen & Segerstedt, 2009). Another component of inventory, *Anticipation stock,* is accumulated in industries that face demand seasonality (Toelle & Tersine, 1989). Such inventories are accumulated prior to periods of peak demand.

Safety stock, on the other hand, is a portion of inventory kept to deal with the variation and uncertainty of supply and demand sources (Tomlin, 2006). The level of safety stock is a function of the degree of uncertainty in those sources, the desired level of customer service specified by the management and the associated costs. The importance of safety stock and its accurate calculation have become more important over time for at least two reasons. (1) With the lean manufacturing movement, small lot production and delivery, cycle stocks and pipeline stocks have decreased, thereby, increasing the chance of stock out (Domingo & Alvarez, 2007; Balakrishnan, Bowne, & Eckstein, 2008); (2) concern for bottom line and managers' awareness of the underestimation of inventory carrying cost have urged them to reduce inventories to the bare minimum. Under such circumstances, accurate calculation of safety stock naturally becomes more important (Louly & Dolgui, 2009; Hou, Zeng, & Zhao, 2009).

Silver et al. (1998) mentions five classes of criteria for determining the level of safety stock: through the use of a common factor, by considering shortage costs, based on service considerations, based on the effects of disservice on future demand, and based on aggregate considerations. In retailing, wholesaling and in some manufacturing settings, the level of safety stock for individual items is incorporated in different inventory control policies to formulate a decision rule for order timing and order quantity for the item. There are basically four common inventory control systems (and several variations of them): order-point, order-quantity (s,Q), which is the subject of this article; order-point, order-up-to-level (s,S); periodic-review, order-up-to-level (R,S); and periodic-review, order-point, order-up-to-Level. (R,s,S) (Buxey, 2006). The explanation of notation is presented in Appendix A.

Most inventory control systems assume that the demand volume can reasonably be modeled by a normal distribution. Other theoretical probability functions such as Geometric (Carlson,1982), Negative Binomial (Deemer, Kaplan, & Kruse, 1974; Ehrhardt, 1979), Poisson (Archibald, 1976; Archibald & Silver,1978), or density functions such as Gamma (Burgin, 1975; Das, 1976), Lognormal (Presutti & Trepp, 1970), Exponential (Brown, 1977), Logistic (Fortuin, 1980), Weibull (Shah, Shah, & Wee, 2009), and many more, have also been suggested by researchers in order to model demand or forecast error. However, due to model complexity associated with employing distributions other than the normal distribution, practitioners have largely preferred the latter one. For example, studies show that the effect of using demand distributions, other than normal, on inventory decision rules is usually small (e.g., Fortuin, 1980).

In formulating inventory decision rules, it has commonly been assumed that the true mean and standard deviation of demand volume are known. However, these parameters are seldom known and usually are estimated from sample historical data. These estimates are subject to sampling variation and hence are random variables. Inaccurate estimates of demand and supply parameters will increase costs such as stock-out and carrying costs. Therefore, robust demand and supply estimates may reduce inventory costs (Jacob & Wagner, 1989). The effects of sampling variability on the estimates of parameters are well established in the field of statistics (Kutner, Neter, Nachtsheim, & William, 2004; Stapleton, 2009; McCloskey, 2009). However, the authors have not yet encountered any publication that *theoretically* addressed this issue in the context of inventory control policies and the computation of safety stock. The existing literature has largely employed ad hoc simulation studies to address the sampling variation of these estimates as described.

A simulation study performed by Ehrhardt (1979) concluded that in a (R, s, S) system with setup, shortage, and holding costs, sample estimates of parameters did not seriously affect the optimal policy. Vaughn (1995) through simulation analysis, showed that in (s, Q) systems when sample estimates of mean and standard deviation of demand during lead time are used, the experienced service level had greater variability around its theoretical or *target* service level than if the true values of the parameters are used. Hojati (1996) suggested the use of the t distribution (instead of the standard normal distribution) to partially rectify this problem. Through simulation analysis, he showed that for a sample of size n=15 periods and a nominal (target) service level of 95%, the average realized service level over a large number of experiments was about 93% when a *z*-score was used. Under the same scenario, he showed that the average realized service level was about 94% when a *t* score (from a t-distribution with n-1 degrees of freedom) was applied. Newer simulation, inventory, or supply chain studies such as the ones by Lee (2008), Mahamani, Rao, and Pandurangadu (2008), Shi and Xiao (2008), Chandra and Grabis (2009), Xiao, Luo, and Jin (2009), Muñoz and Torres (2009), Wang and Prabhu (2009), and Kattan and Khudairi (2010) do not address the effect of sample variability.

In this paper we discuss the shortcomings of the basic traditional inventory models as related to the effect of sample variability on the estimates of parameters of the distribution of demand during lead time. We propose improved formulations that directly incorporate the sampling variation in these estimates. The proposed model is pragmatic and is easy to use. The paper will continue by discussing the basic traditional inventory models and then offers improved formulations for the (s, Q) system when the replenishment lead time is constant. We then compare the proposed and the traditional models in achieving the target service level through numerical demonstrations and from a theoretical standpoint. Finally, the overall findings of the research will be summarized and potential extensions to this research are proposed.

THE BASIC TRADITIONAL INVENTORY MODEL

In this section, we exclusively consider the continuous review order-point order quantity policy (s, Q). The (s, Q) inventory model offers one of the simplest and most widely used decision rules among inventory control models. According to the policy of this model, an order of size Q is placed when the on-hand inventory reaches a predetermined level, s (the order point). The assumptions underlying this model are (Silver et al., 1998; Buxey, 2006):

1. Demand per unit time, d, is a random variable with stationary mean and variance. In addition, the demands in different time periods are independent.
2. A replenishment order of size Q is placed at the moment the inventory position reaches s. This implies that the demand transactions are of unit size (or small quantity). In addition, the value of Q is predetermined and is independent of s.
3. The demand per unit time follows a normal distribution with parameters μ_d and σ_d. In turn, the forecast error has a mean of zero and a standard deviation of σ_d.
4. It is assumed that shortage costs are high enough so that a high service level (a low probability of stock out) should be maintained.

Based on these assumptions, the demand (D) during a fixed lead time (Ld) is Ld and follows a normal distribution with parameters, $L\mu_d$, and $\sqrt{L}\sigma_d$. In the simplest case when a probability of a stock out, p , or a *target* service level of $SL^* = 1 - p$ is specified, the order point, s, is determined as follows (The reader may refer to Appendix A for notation used):

Given,

$$P(D \le s) = SL^*$$

The random variable,

$$\frac{D-\mu_D}{\sigma_D}=Z \tag{1}$$

follows a standard normal distribution. A safety factor Z_{SL^*} is determined such that $\Phi(Z_{SL^*})=SL^*$, where $\Phi()$ is the cumulative distribution of the standard normal. The minimum order point (s) is computed as follows:

$$\frac{s-\mu_D}{\sigma_D}=Z_{SL^*}$$

$$s=\mu_D+Z_{SL^*}\sigma_D \tag{2}$$

This model produces an expected service level (over large number of items) equal to SL^*. The quantity $k\sqrt{L}\sigma_d$ is the amount of safety stock.

The formulation assumes that the true value of parameters $L\mu_d$ and $L\mu_d$ (or μ_D and σ_D when one deals with the distribution of demand during lead time, directly) are known. However, in practice the true value of these parameters are seldom known and only estimates of those parameters from a sample of size n (most recent demands per period or most recent lead times) are available. It appears that, traditionally, researchers and practitioners have advocated the use of the following order point model (we refer to it as Model s):

$$s=\hat{\mu}_D+Z_{SL^*}\hat{\sigma}_D \tag{3}$$

It is obvious that the average realized service level, SL , when estimates are used can deviate from the target service level, SL^*. The magnitude of the deviation will potentially be larger when the sample size, n, is also small.

THE PROPOSED MODEL

As mentioned before, the basic traditional model, $s=\mu_D+Z_{SL^*}\sigma_D$, assumes that the mean and standard deviation of demand during *lead time* for each item, μ_D and σ_D, (or demand per unit time, μ_d and σ_d) are known quantities. In practice, these parameters are usually estimated by the sample mean and standard deviation from the most recent observed demand data for each item. The sample mean, $\hat{\mu}_D$, and sample standard deviation, $\hat{\sigma}_D$, are computed as follows:

$$\hat{\mu}_D=\frac{\sum_{i=1}^{n}D_i}{n} \tag{4}$$

$$\hat{\sigma}_D=\sqrt{\frac{\sum_{i=1}^{n}(D_i-\hat{\mu}_D)^2}{n-1}} \tag{5}$$

Where, n is the number of most recent lead times considered. Substituting (4) and (5) for the true mean and standard deviation in (2) introduces two sources of variability, one due to the use of sample mean and the other due to sample standard deviation. We present two intermediate cases in order to make the proposed model clear.

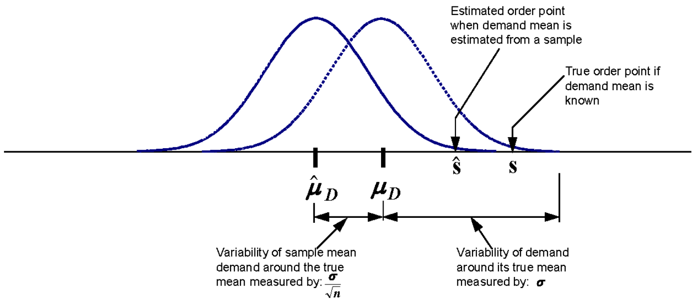
Case 1. The Proposed Model: μ_D Unknown

When σ_D is known and only $\hat{\mu}_D$ is calculated from sample data and is substituted for μ_D, the distribution of $D-\hat{\mu}_D$ is approximately normal with the following parameters:

$$E(D-\hat{\mu}_D)=0 \tag{6}$$

$$Var(D-\hat{\mu}_D)=\sigma_D^2\left(1+\frac{1}{n}\right) \tag{7}$$

Figure 4. An example of the sample estimate of the demand distribution location versus its unknown true location



The derivation of (7) is shown in Appendix B. The variance component in (7), as illustrated in Figure 4 in the Appendix, is the sum of the variances from two sources:

1. Variance within the probability distribution of the demand during lead time (σ_D^2), and
2. Variance in possible location of the distribution of the demand during lead time (σ_D^2 / n)

Based on (6) and (7), equation (1) can be restated as:

$$\frac{D - \hat{\mu}_D - 0}{\sigma_D \sqrt{1 + \dfrac{1}{n}}} = Z$$

And the minimum order point is defined by Model s_1 :

$$s_1 = \hat{\mu}_D + Z_{SL^*} \sigma_D \sqrt{1 + \frac{1}{n}} \qquad (8)$$

This treatment of the sample variability of mean, as outlined above, is often termed a "prediction interval" in statistics, and is used to create a confidence interval estimate of the value of a randomly occurring individual item from a normal population (Kutner, Neter, Nachtsheim, & William, 2004; Stapleton, 2009; McCloskey, 2009). It is most commonly used in *regression analysis*. However, to our surprise, the concept has not been addressed in applications such as the inventory control models.

The quantity $SS_1 = Z_{SL^*} \sigma_D \sqrt{1 + 1/n}$ is the amount of safety stock generated by this model. Clearly, the safety stock (SS_1) computed under model s_1 is greater than the safety stock (SS) computed under model s . One can conclude that the average service level (SL_1), realized under model s_1 , is greater than the average service level (SL) realized under Model s . It is important to note that neither SL nor SL_1 are necessarily equal to SL^*.

In order for Model s to produce a service level similar to Model s_1 , its safety stock should be corrected by the factor SS_1 / SS. This ratio can be simplified as follows:

$$\frac{SS1}{SS} = \frac{Z_{SL^*} \sigma_D \sqrt{1 + \dfrac{1}{n}}}{Z_{SL^*} \sigma_D} = \sqrt{1 + \frac{1}{n}} \qquad (9)$$

or

$$SS_1 = SS \sqrt{1 + \frac{1}{n}}$$

Thus, model s is equivalent to model s_1 and produces the same average service level

SL_1 if its safety factor, Z_{SL^*}, is corrected by the factor, $\sqrt{1+\frac{1}{n}}$.

Case 2. The Proposed Model: σ_D **Unknown**

When μ_D is known and $\hat{\sigma}_D$ is estimated from sample data, the distribution (1) does not follow a normal distribution, but the quantity $\frac{D-\mu_D}{\hat{\sigma}_D}$ follows *t* with *n-1* degrees of freedom. The minimum order point is defined by Model s_2:

$$s_2 = \mu_D + t_{n-1,SL^*}\hat{\sigma}_D \tag{10}$$

The quantity $SS_2 = t_{n-1,SL^*}\hat{\sigma}_D$ is the amount of safety stock generated by this model. The safety stock (SS_2) computed under Model s_2 is larger than the safety stock (SS) computed under Model s since $t_{1-p} \geq Z_{1-p}$ for all $1-p > 0.5$. One can conclude that the average service level (SL_2) realized under Model s_2 is greater than the average service level (SL) realized under Model s. The safety stock correction factor can be computed as follows:

$$\frac{SS_2}{SS} = \frac{t_{n-1,SL^*}\hat{\sigma}_D}{Z_{SL^*}\sigma_D} = \frac{t_{n-1,SL^*}}{Z_{SL^*}} \tag{11}$$

or

$$SS_2 = SS\left(\frac{t_{n-1,SL^*}}{Z_{SL^*}}\right)$$

Thus, model s is equivalent to model s_2 and produces the same average service level SL_2 if its safety factor, Z_{SL^*}, is corrected by the following factor: $\frac{t_{n-1,SL^*}}{Z_{SL^*}}$.

Case 3. The Proposed Model: μ_D **and** σ_D **Unknown**

This case represents the most realistic environment. It is not difficult to show that under such conditions, the order point is calculated by the proposed model, Model s_3:

$$s_3 = \hat{\mu}_D + t_{n-1,SL^*}\hat{\sigma}_D\sqrt{1+\frac{1}{n}} \tag{12}$$

The quantity $SS_3 = t_{n-1,SL^*}\hat{\sigma}_D\sqrt{1+1/n}$ is the amount of safety stock generated by this model. Obviously, SS_3 is greater than SS, SS_1, and SS_2. As a result, the average realized service level, SL_3, is greater than SL, SL_1, or SL_2, and is equal to SL*. The safety stock correction factor can be computed as follows:

$$\frac{SS_3}{SS} = \frac{t_{n-1,SL^*}\hat{\sigma}_D\sqrt{1+\frac{1}{n}}}{Z_{SL^*}\hat{\sigma}_D} = \frac{t_{n-1,SL^*}}{Z_{SL^*}}\sqrt{1+\frac{1}{n}} \tag{13}$$

or

$$SS_3 = SS\left(\frac{t_{n-1,SL^*}}{Z_{SL^*}}\sqrt{1+\frac{1}{n}}\right)$$

Thus, model s is equivalent to model s_3 and produces the same average service level, SL_3, if its safety factor, Z_{SL^*}, is multiplied by the following safety stock correction factor (SSCF):

$$SSCF = \frac{t_{n-1,SL^*}}{Z_{SL^*}}\sqrt{1+\frac{1}{n}} \tag{14}$$

Table 1. Actual vs. nominal service levels for selected sample sizes

	Actual Service Levels		
Sample Size (n)	Nominal SL = .90	Nominal SL = .95	Nominal SL = .99
5	0.8465	0.8962	0.9495
10	0.8736	0.9244	0.9731
20	0.8869	0.9375	0.9825
30	0.8913	0.9418	0.9852
60	0.8956	0.9459	0.9877
100	0.8974	0.9476	0.9887

NUMERICAL DEMONSTRATIONS

The traditional method of calculating the reorder point ($s = \hat{\mu}_D + Z_{(SL^*)}\hat{\sigma}_D$) results in levels of safety stock which yield actual service levels which are below the nominal (target) service levels on average. Table 1 shows the actual average service levels achieved for nominal (target) service levels of 90%, 95%, and 99% based on selected sample sizes.

Figure 1 shows graphically the actual service levels with sample sizes of n= 2 through n = 30 periods for each of the three nominal service levels.

Clearly, as the sample size increases, the actual service level approaches the target level. However, relatively small samples of time periods are typically used in order to reflect recent demands. Use of the proposed model corrects the calculation of safety stock and makes the actual service level equal, on average, to the nominal level.

The traditional model for calculating reorder point and safety stock becomes equivalent to the proposed model, Model s_3, by multiplying the following safety stock correction factor (SSCF). A graph of safety stock factors is shown in Figure 2 for sample sizes of n = 2 through n = 30 and service levels of 90%, 95%, and 99%. For n = 2 and a 99% service level, the factor is off the graph at a value of 16.753. Selected numerical values of SSCF are shown in Table 2.

Table 2 shows that for a nominal service level of 95%, safety stock (as calculated by the traditional method) must be increased by 16.9% if a sample of n = 10 is used and by 5% if a relatively large sample of n = 30 is used. As the sample size increases, this correction factor approaches 1 (and becomes unnecessary). However, even for a very large sample of n = 100, safety stock must be increased by 1.5% to achieve a nominal service level of 95%. Typically, relatively small samples of time periods are used in order to reflect recent demand. Thus, it is clear that the traditional method results in significantly understated levels of safety stock. The effects of (traditional) Model s, Models s_1 and s_2, and the proposed Model s_3 are shown graphically in Figure 3, under the assumption of a nominal service level of 95%.

Figure 3 shows: (1) the average actual service level provided by the basic traditional model, (2) the service level which would be realized if the model were corrected only for errors in estimating the mean, (3) the actual service level realized by correcting only for errors in estimating the standard deviation, and (4) the average service level realized by the proposed model, which corrects for both errors. Clearly, all three alternative models perform better than the basic traditional Model s. Only Model s_3 (i.e., the Proposed Model) on average achieves the nominal service level, SL*, with small sample sizes.

Figure 1. Actual vs. nominal Service Levels (SLs) based on traditional safety stock model, s

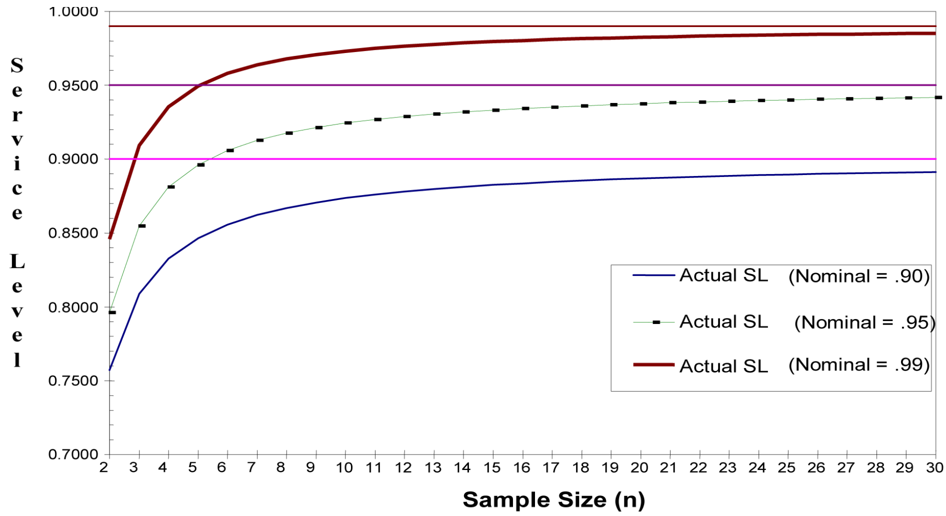


Figure 2. Factors required correcting safety stock calculated by the traditional models in order to achieve target level

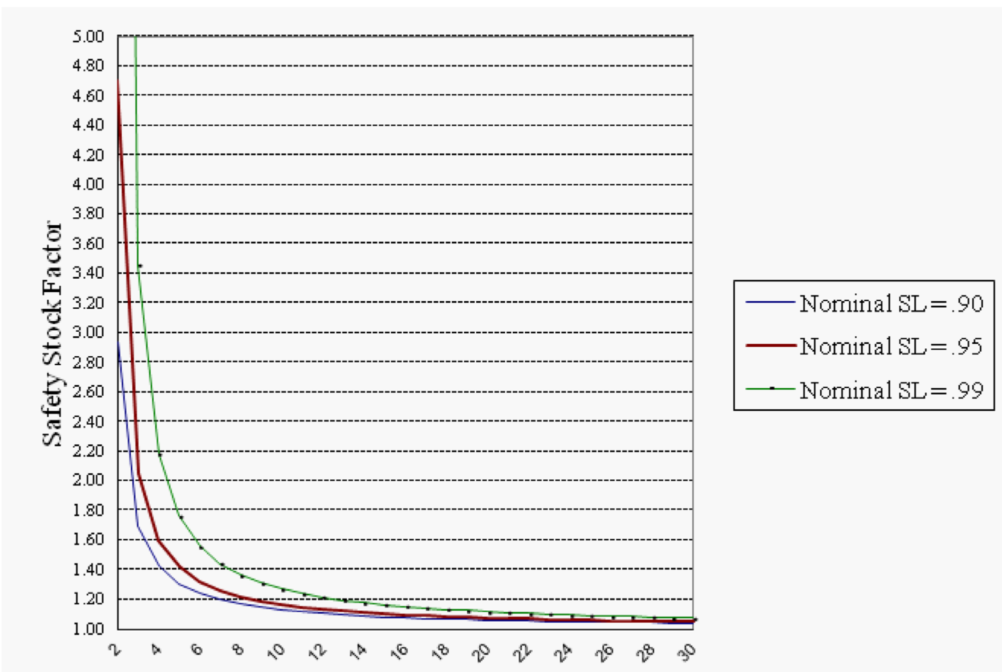


Table 2. Safety stock factors to achieve nominal service levels for various sample sizes

Sample Size (n)	Nominal SL = .90	Nominal SL = .95	Nominal SL = .99
5	1.311	1.420	1.764
10	1.132	1.169	1.272
15	1.084	1.106	1.165
20	1.062	1.077	1.117
30	1.040	1.050	1.076
60	1.020	1.024	1.036
100	1.012	1.015	1.022

FURTHER DISCUSSION AND CONCLUSIONS

The importance of safety stock and its *accurate* calculation has become more important over time. However, the inventory models that have commonly been suggested in the literature could not produce (theoretically) accurate policies. A sound inventory model should incorporate many factors that characterize demand. Factors that characterize demand such as demand distribution, mean, and variance are seldom known, thereby are approximated or estimated from sample historical data. This paper focused on two sources of variability that are present when sample estimates of mean and standard deviation, instead of the true but unknown parameters, are used.

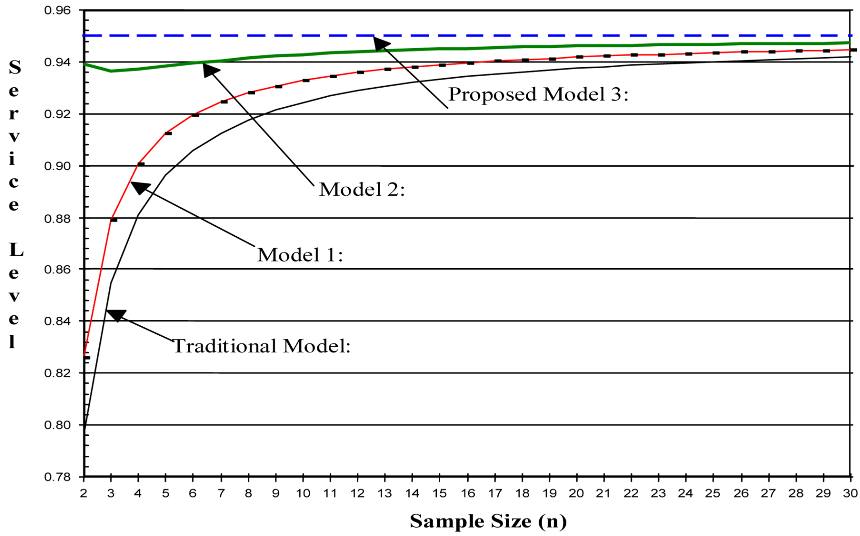
It is noticeable that the treatment of sampling variability of sample mean has not gained attention in inventory management literature. On the other hand, the treatment of sampling variability was addressed with a limited scope through ad hoc simulation studies (Hojati, 1996). This paper addressed the shortcomings of the basic traditional inventory models as related to sampling variations and indicated that these models, in general, understated the required safety stocks.

The proposed model has shown that the service level realized under the basic traditional models was on average smaller than the target or desired service level, especially when small sample size was used. We have developed several improved models that directly incorporated the sampling variations of estimates. Through numerical demonstrations, the service level realized by the traditional model was compared with several proposed models. The proposed model that incorporates both the variability of sample mean and standard deviation produces an average service level over many items (SL_3) that is equal to the target or nominal service level, SL^* .

We believe that the result of the proposed model is of special practical importance. In today's competitive global supply chain systems, inventory managers at every stage is trying to reduce the impact of bullwhip effect and thus must be very attentive to the true downstream demands (Kristianto & Helo, 2009) with minimum inventory costs (Mahamani, Rao, & Pandurangadu, 2008) to maximize profit (Shi & Xiao, 2008; Ma, 2008). Inventory managers, who are not aware of the potential effect of sampling variation of estimates on the realized service level, can face higher inventory related costs and lower customer satisfaction. Without being aware of the potential effect of sampling variation, managers may not be able to safeguard (e.g., applying a larger sample size) against potential unwarranted results.

Three decades ago, some researchers (e.g., Silver, 1981) referred to the control related cost as being a major reason for *assuming* that the true parameters of demand are known. In our opinion, the control related costs should be substantially less than the inventory related

Figure 3. Actual service level achieved by various models when the target service level is 95%



costs or the cost due to customer dissatisfaction because of the availability of information technology such as the Internet (Yan & Ghose, 2008) and automatic data capture technologies such as bar coding (Kattan & Khudairi, 2010) and RFID (Mondragon, Lyons, Michaelides, & Kehoe, 2006; Ozelkan & Galambosi, 2008; Soon & Gutiérrez, 2008). With a little additional complexity, any of the three cases of proposed model explained in earlier sections, produce results that are theoretically better than the basic traditional model. Therefore, we highly recommend the use of the proposed Model s_3 by managers because it is only marginally more complex than the traditional model but produces superior solutions – customer service levels that on average are equal to the target service levels.

The procedure of this paper can be extended to other models such as (R,S), (R,s, S), and so on. A similar approach can be undertaken to improve those inventory models that also treat the lead-time (L) as a random variable. Future research may also include comparing the proposed model with other models in the literature in the quality of solutions they produce. Another extension of this research may consider demand distributions such as Poisson, binomial or uniform.

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APPENDIX A

Notation used in the paper

d_t	Demand per unit time (a stationary random variable)
μ_d	Expected total demand per unit time (population)
σ_d	Standard deviation of total demand per unit time (population)
$\hat{\mu}_d$	Sample mean demand per unit time
$\hat{\sigma}_d$	Sample standard deviation of demand
D	Demand during lead time (a stationary random variable)
μ_D	Expected total demand during lead time
σ_D	Standard deviation of demand during lead time
$\hat{\mu}_D$	Sample mean demand during lead time
$\hat{\sigma}_D$	Sample standard deviation of demand during lead time
L	Replenishment lead time (a constant)
SS	Safety stock, in units
s	Order point, in units
Q	Order quantity, in units
SL^*	Target, nominal or desired service level
SL	Realized service level
$\Phi()$	Cumulative distribution of the standard normal
R	Review period
S	Maximum inventory position

APPENDIX B

Derivation of the Components of Variance

$$Var(D - \hat{\mu}_D)$$

$$= Var(D) + Var(\hat{\mu}_D) - 2Cov(D, \hat{\mu}_D)$$

Assuming the true standard deviation of demand per unit time (σ^2_D) is known and since the demand in different time periods is independent, the covariance term drops and the variance equation becomes:

$$Var(D - \hat{\mu}_D) = \sigma^2_D + \frac{\sigma^2_D}{n}$$